Non-Classical Logics and Applications

Itala M. Loffredo D'Ottaviano

The Group for Applied and Theoretical Logic
Centre for Logic, Epistemology and the History of Science (CLE)
Department of Philosophy (IFCH)
State University of Campinas - UNICAMP
itala@cle.unicamp.br

Logic, if considered as the science of the deductive reasoning, studies the consequence relation, treating of the valid inferences, that is, the inferences whose conclusions have to be true when the premises are true.

Therefore, logic can be considered as 'the study of reason', 'the study of reasoning'. So, the aim of logic consists of the mention and the study of the principles used in the deductive reasoning.

Under this conception, logic is called *deductive logic*.

However, we can consider another logic, the *inductive logic*, that does not treat of the valid inferences, but treats of the probable inferences.

Let us consider for instance, the following arguments:

- The sun has been born every day.
  Then, the sun will be born tomorrow.

- 80% of the interviewed people will vote the candidate X.
  Then, 80% of all the electorate will vote X.

- This vaccine worked well in monkeys.
  This vaccine worked well in pigs.
  Then, this vaccine will work well in human beings.

Such arguments are not deductive. Even if the premises are true, they do not logically imply the conclusion; however, if the premises are true, then the conclusion is plausible, that is, it is probably true.

Yet we have another kind of logic, the *abductive logic*, that has been reconsidered during the last decades. Charles Peirce, who coined the term *abduction*, argued that the true reasoning in science, in special in the empirical sciences, is the *abductive reasoning*. (see   ).

Contemporaneously, logic has been transformed into a mathematical discipline, the *mathematical logic*, with special characteristics – deductive, it is the study of the type of reasoning done by mathematicians.

The contemporary logicians build appropriate *artificial languages* to deal with the consequence relation. Such languages have two relevant dimensions: the syntactic dimension and the semantical dimension.

In order to work in a *formal theory*, it is necessary to explicitate its *language*. That is, its symbols and the combination rules to which they are submitted for the construction of the *terms* and the *formulae* of the language.

The *axioms* and *deduction rules* of the theory are specified among the well-formed formulae of the language. From the axioms and by using the deduction rules, the *theorems* of the theory are proved (see Schoenfield (1967)).

The combinatory dimension of a language is called its *syntactic* dimension.

The *semantical dimension* of a language takes into consideration the extra-linguistic objects to which the symbols and expressions of the language refer to, and their meaning. It deals with the concepts of *structure, validity* of formulae and *model*.

So, contemporary theories, constructed over languages, axioms and deduction rules, are constituted by the formal theory (its axiomatics) and by its semantics.

The results relative to the *completeness, consistency* and *decidability* of a theory are important meta-theorems which establish the relations between these two dimensions.

1. Some historical comments

Until the beginning of the 20th Century, there was a unique logic, pure, formal or theoretical, founded by Aristotle (384-322 b.C.), and whose most important systematizer was Frege (1848-1925).

Most of Aristotle's relevant contribuition to logic is in the group of works known by ***Organon***, especifically in the ***Analytica Priora*** and in ***De Interpretatione*** (see Aristotle (1978a) and (1978b)). Aristotle created the *theory of sylogisms* and axiomatized it. He also initiated the development of modal logic.

The theory of sylogisms constitutes the first formal system of the history of logic. Contemporaneously, we can interpret it as a fragment of the first-order predicate calculus.

It seems that logic has been an exclusive product of the occidental culture.

The Arabs, in spite of the relevance of their contributions to mathematics, in special to the development of algebra, contributed almost nothing to logic; the Indian logic and the Chinese logic, though some specific characteristics, were not significant comparatively to the Greek logic.

During the medieval period, the Aristotelian logic was largely studied, modal logic was very discussed and we may mention in particular the work developed by the scholastics. However, we cannot consider that creative conceptions and works have stimulated the development of logic.

During the Renaissance period, in spite of its importance to the west civilization, nothing relevant to the improvement of logic was created.

Modern logic initiated in the 17th Century with Gottfried Wilhelm Leibniz (1666) and began developing in partnership with mathematics (see D'Ottaviano and Feitosa (2003)). Leibniz' programme looked for the construction of a *universal* *language*, based on an *alfhabet of thought*. This language would propitiate a fundamental knowledge of everything.

Leibniz added to his work the project of the construction of a *calculus ratiocinatur*, that is, a calculus of reason. In spite of Leibniz' programme being not theoretically executable, his *calculus ratiocinatur* constitutes an important forerunner of the methodology of contemporary logic.

However, most of Leibniz' contributions to logic remained not published during his life and unknown until the beginning of the 20th Century, when Couturat publishes the ***Opuscules et fragments inédits de Leibniz*** (see Couturat (1903)).

Immanuel Kant contributed very little to logic. But, in the Preface of his ***Kritik der reinen Vernunft***, edition of 1787, he explicitly declares that logic had not given any important step, neither ahead nor behind, since Aristotle – so, it seemed to be a complete and finished science (Kant (1787)).

During the 19th Century, among the forerunners of contemporary logic we may mention Boole, de Morgan, Schröder and Peirce (see Boole (1847), De Morgan (1847) and Bochenski (1961)).

The true founder of modern logic was Gottlöb Frege.

In 1879, the essential steps for the introduction of the logistic method were given by Frege in his ***Begriffsschrift***, this book containing, for the first time, the propositional calculus in its modern logistic form (Frege (1879)). Frege introduces the distinction between *language* and *meta-language*.

In 1884, Frege addopts the thesis – *logicism* – that arithmetics is a branch of logic, in the sense that every term of arithmetics can be defined from the logical terms and that every theorem of arithmetics can be proved from the logical axioms.

In 1874, George Cantor creates the *set theory*. He publishes his first work on a new theory of the infinite, where a collection of objects, even infinite, is conceived as a complete entity. For Cantor, a set was intuitivelly a collection of elements that satisfy a given property (see Cantor (1874)).

At the beginning of the 20th Century, this apparently naïve acceptance of any collection as being a set, propitiated the appearance of paradoxes in the foundations of the nascent set theory. Some of the known paradoxes related to set theory are the Russell Paradox, Cantor Paradox and the paradox of the biggest ordinal; some semantic paradoxes were also presented and some old known paradoxes were rediscussed, as for instance the liar paradox and the barber paradox.

From Cantor Paradox, relative to the biggest cardinal number, Russell obtained the famous Russell Paradox and communicated it to Frege in 1902.

In 1908, in the opening section of the "IV International Congress of Mathematics", hold in Rome, Poincaré claimed the mathematical community to find a solution to the paradoxes crisis, that seemed to shake the foundations of mathematics (see ***Atti del IV Congresso Internazionale dei Matematici*** (1909)).

Zermelo and Russell were already working, looking for the fundamental principles that could underlie a consistent set theory, without contradictions.

Russell and Whitehead publish their ***Principia Mathematica*** *in* 1910, 1912 and 1913. They introduce the *ramified type theory,* a system that establishes a hierarchy of types and collections (Whitehead and Russell (1973)).

Type theory presents a general solution to the problem of the paradoxes.

Set theory, nascent at the beginning of the 20^{th} Century, resisted to the crisis and set theories present a partial solution to the problem of paradoxes, constituting strong systems for the foundation of mathematics.

2. The Aristotelian Classical Logic

The *Aristotelian classical logic*, in its elementary part, essentially deals with the logical connectives of negation, conjunction, disjunction, implication and equivalence; with the existencial and universal quantifiers and with the equality predicate; and deals with some of their extensions, such as the higher-order predicate calculus or some usual systems of set theory, as for instance Zermelo-Fraenkel, von Neumann-Bernays-Gödel, Tarski-Morse-Kelley or Quine's NF theories. It is characterized as a logic of *propositions*.

From Frege's work, classical logic got an extense, consistent and almost definitive form in the ***Principia Mathematica*** of Whitehead and Russell.

In its contemporary status it is powerful and contains all the old Aristotelian sylogistic, conveniently reformulated. Besides, in a certain sense, traditional mathematics can be reduced to classical logic, for it can be definible from the idea of set.

In its contemporary clothes, logic is considered as a deductive formal system, built over a formal language, that would have the charge of eliminating interpretative doubts (see, for instance, Kleene (1952) and Schoenfield (1967)).

The basic elements of the *classical first-order predicate calculus* with equality, $\mathbf{CQC}^{=}$, are the *propositions*, that are expressions that admit a *truth value*: (0) *false* or (1) *true*.

From the *simple* or *atomic propositions* we can form the *compound* or *molecular propositions*, by using the linguistic connectives and quantifiers, as for instance the symbols $\neg$ (for negation), $\wedge$ (for conjunction), $\vee$ (for disjunction), $\rightarrow$ (for implication), $\leftrightarrow$ (for bi-conditional), $\forall$ (for the universal quantifier), $\exists$ (for the existence quantifier) and = (for the predicate of equality).

The classical propositional calculus **CPC** corresponds to a subsystem of the classical quantificational calculus with equality $\mathbf{CQC}^{=}$, whose language does not contain the symbols relative to the quantifiers, "$\forall$" and "$\exists$", and the equality predicate symbol "=", and whose axioms and deduction rules do not contemplate the quantifiers and the equality predicate.

A simple and intuitive way of attributing a semantics to **CPC** is through functions of valuation. Given a valuation v, there is a unique way of extending v to all the formulae A of the language of **CPC**, by respecting the following tables for every one of the propositional connectives:

A	$\neg A$
0	1
1	0

A	B	$A \vee B$	$A \wedge B$	$\alpha \rightarrow \beta$	$\alpha \leftrightarrow \beta$
0	0	0	0	1	1
0	1	1	0	1	0
1	0	1	0	0	0
1	1	1	1	1	1

A formula A of **CPC** is said to be *valid*, or a *tautology*, if, for every valuation v, we have that $v(A) = 1$. As examples, let see that the formulas $\neg(A \wedge \neg A)$ and $(A \vee \neg A)$ are tautologies:

A	$\neg A$	$A \wedge \neg A$	$\neg(A \wedge \neg A)$	$A \vee \neg A$
0	1	0	1	1
1	0	0	1	1

The *formal system* **CQC**$^{=}$ is determined by *axioms* and *deduction rules* (or *inference rules*), among these the Rule of Modus Ponens: *If A and B are formulae, then from A and* $A \rightarrow B$ *we obtain B*.

The basic principles of the first-order predicate calculus, known as the *basic laws of the Aristotelian thought,* are the following:

- *Principle of (Non-)Contradiction;*
- *Principle of the Excluded Middle;*
- *Reflexivity of Identity.*

3. Non-Classical Logics

Mathematics in the 19th Century – one of the golden periods of mathematics – strongly influenced the culture and thought of the 20th Century, directly or indirectly contributing to the arising of mathematical logic and of non-classical logics in general. One of the fundamental marks was the appearance of the non-Euclidean geometries.

During part of the 19th Century, the French mathematicians conceived mathematics as being of an intuitive nature, a kind of physical science. The German mathematicians, mainly from Cantor's influence, conceived mathematics as a purely abstract science.

The 19th Century transmitted us the Cantorian abstract vision and the French concrete vision of Poincaré, Lebesgue and others, contemporary mathematics and logic being a kind of synthesis of the French and German positions.

The usual set theory and Russell's type theory, over which arithmetics can be founded – and so all the traditional mathematics –, maintain classical logic with its basic principles, the basic laws of the Aristotelian thought, as their underlying logic.

However, the paradoxes of set theory and some non-solved questions relative to the concept of infinite still left to the logicians several problems concerning the foundations of mathematics.

Already at the end of the 19th Century some pioneer works, looking for non-Aristotelian solutions for some logical questions, were forerunners of non-classical logics in general.

At the first decades of the 20th Century, several philosophers and mathematicians, motivated by distinct questions and objectives, created new logical systems, distinct of the Aristotelian logic.

We can assert that non-classical logics differ of classical logic for:

i) They can be based on languages richer in manners of expression;

ii) They can be based on completely distinct principles; or

iii) They can have a distinct semantics.

The first of these arises, for example, with the usual modal logics, which are obtained from classical logics by enriching their language through the introduction of certain intentional operators to express the concepts of necessity and possibility. The same is true of some forms of deontic logics, where operators expressing obligation and prohibition are introduced, and of temporal logics. Furthermore, it happens that these enriched logics require a distinct semantical treatment.

The second case arises, for instance, with intuitionistic logic, in which the law of excluded middle of classical logic is dropped. It also happens that a radically different semantical point of view is necessary. Something very similar holds for paraconsistent logics, in which the principle of (non-)contradiction is not valid in general.

Evidently, those three main modes in which a heterodox logic may differ from classical logic are not independent of each other. Thus, dropping some law of classical logic may accordingly require semantical alterations – this is to be expected if any kind of completeness of the logic is to be required.

Generally speaking, according to Haack (1974) we can consider that there are two principal categories of non-classical logics:

1. Logics which are presented as *complementary* to classical logic, and

2. Logics which are posed as *alternative* to classical logic, also called *deviant* or *heterodox logics*.

Complementary logics do not infringe the basic principles of classical logic, but they only enlarge, widen and complement its scope.

We proceed to mention some of the most important complementary non-classical logics:

a) *Modal logics*

b) *Temporal logics*

c) *Deontic logics*

d) *Epistemic logics*

e) *Erotetic logics*

f) *Imperative logics*

g) *Other intensional logics*

h) *Variable binding term operators* (such as, for example, Hilbert's ε-symbol or the descriptor symbol) *logics*

i) *Conditional* (as for instance the counterfactual) *logics*.

Heterodox logics, rivals of classical logic, were conceived as new logics posed as alternative to classical logic, determined to substitute classical logic in some domains of knowledge. They collapse basic principles of classical logic.

The *paracomplete logics* are the logics in which the Principle of the Excluded Middle is not valid. As examples we may mention the *intuitionistic logics* and the *many-valued logics*.

In p*araconsistent logics* and *relevant logics* the Principle of (Non-)Contradiction may be not valid, in general.

There exist several other kinds of alternative non-classical logics, as for instance *linear logic, fuzzy logic* and *non-monotonic logic*.

Obviously, a logic cannot be classed in an absolute fashion as complementary or alternative.

Contemporary logic has very much evolved and relatively to certain types of logic, some of them which profoundly differ from the classical in their syntax or semantics, it would be difficult to classify them in either of the presented two categories.

We observe that the foundations of non-classical mathematics (for example, modal arithmetic, relevant arithmetic and paraconsistent set theory) may be included in the field of non-classical logic.

Moreover, inductive inferences may be systematically and formally studied and such studies could be viewed as inductive logics. If this is so, inductive logics could be classified, at least in part, among non-classical logics. The usual domain of non-classical logics would then be considerably extended considering the number of problems related to techniques of inductive inference.

The same approach may be considered relatively to abductive reasoning and abductive logics.

The creation of non-classical logics, in particular of heterodox logics, has very naturally originated deeply interesting and important philosophical problems. The very possibility of posing heterodox logics as "true" logics can lead to such problems.

There are authors who, like Quine, think that in trying a change of logic we are really changing the subject and no longer speaking of Logic proper, giving no room for the existence of heterodox logics (see Haack (1974)). Nevertheless, it can be argued that even given such a change of subject, the subject continues to be one of Logic.

Moreover, we can observe that some of the heterodox logics, as for instance paraconsistent logics, although profoundly diverging from classical systems, can be used as an alternative in solving problems in all situations where the latter can be so used. Here, the situation is entirely similar to what happens to non-Euclidean geometry, given that some systems can be used in solving the usual geometric problems, for it coincides “locally” with Euclidean-geometry.

We can accept the existence of complementary and alternative logics able to replace classical logic in various specific domains of knowledge. However, one can not deny that much philosophical debate is needed before we reach an understanding of the exact nature of classical laws and of heterodox logics in general, considered as true logics and not only as mere mathematical formalisms.

4. Paraconsistent Logic

A deductive theory, in whose language there is a symbol for negation, is said to be *consistent* if there is not any formula *A* of its language such that *A* and the negation of *A* are theorems; otherwise, the theory is *inconsistent*, or *contradictory*.

A theory is said to be *trivial* if every formula of its language is a theorem.

A *logic* is said to be *paraconsistent*, if it can be used as the underlying logic to inconsistent but non-trivial theories, that are called *paraconsistent theories*.

If the underlying logic of a theory is classical logic, or for instance intuitionistic logic, inconsistency entails triviality, and conversely.

In paraconsistent logics, in a certain sense, the scope of the Principle of (Non-) Contradiction is restricted.

In fact, in paraconsistent logics, the *Principle of (Non-) Contradiction*, in the form

$$\neg(A \land \neg A)$$

is not necessarily non-valid, but , in every paraconsistent logic, from a formula *A* and its negation $\neg A$ it is not possible, in general, to deduce any formula *B*.

That is, in paraconsistent logics *lato sensu*, the following formula is not a theorem, in general:

$$A \to (\neg A \to B).$$

The two forerunners of paraconsistent logic are Jan Łukasiewicz and Nicolai Vasiliev. Both, working independently of each other and inspired by the development of non-Euclidean geometry, claimed in 1910 and 1911 that a revision of the basic laws of the Aristotelian logic would yield new non-Aristotelian systems of logic, in particular systems admitting violation of the Law of (non-) Contradictions.

Łukasiewicz, in 1910, published a paper (in German) and a book (in Polish) titled *On the principle of contradiction in Aristotle*, later translated into English in Borkowski (1970). He shows that the arguments built in order to justify such a law, all derived from Aristotle, are feeble and does not exclude the possibility of non-trivial contradictory theories being true.

Meinong’s theory of objects (see Meinong (1910)) seems to have influenced Łukasiewicz’ work, unless in its initial stage.

Łukasiewicz opened the way for non-classical logic but, in spite of having discussed the general validity of the most fundamental Aristotle’s principle, he did not create paraconsistent logic. Some years later, he introduced the first systems of many-valued logics, as an attempt to investigate the modal propositions and the notions of possibility and necessity closely related to such propositions (see Łukasiewicz and Tarski (1930)).

Post, independently of Łukasiewicz and motivated by formal questions of propositions, also introduced his systems of many-valued logics in 1921 (see Post (1921)).

Vasiliev, on account of his imaginary logics, has been considered a forerunner of many-valued and paraconsistent logic (see Arruda (1977), (1984) and (1992)). In his 4 papers of 1910, 1911, 1912 and 1913 he discusses the derogation of the Law of (non-) Contradiction and the possibility of the construction of a new logic, non-Aristotelian. Vasiliev (1925) presents a 3 pages summary of his ideas.

Bochvar (1939) introduces a three-valued calculus and studies its applications to the analysis of contradictions, but we consider that we can not state that Bochvar proposed a paraconsistent treatment of contradictions.

Stanislaw Jaśkowski, one of the Łukasiewicz' disciples, was the first logician to construct a formal system of paraconsistent logic, in 1948. His main motivations were the systematization of theories which contain contradictions as it occurs in dialectics, the study of theories where there are contradictions caused by vagueness, and the direct study of some empirical theories whose postulates or basic assumptions are contradictory.

Jaśkowski (1948) and (1949) were published in Polish and his first paper in English appeared only in 1969. Despite its importance and possibility of applications, Jaśkowski's discussive logic $\mathbf{D_2}$ was restricted to the propositional level.

Other pioneer works are Nelson (1959) and Asenjo (1966).

The Brazilian logician Newton Carneiro Affonso da Costa may be considered the founder of paraconsistent logic (see Arruda (1980) and (1989)), da Costa and Marconi (1989), D'Ottaviano (1991), Bobenrieth (1996), Guillaume (1996), da Costa et al. (1995)).

In the years 1950, without knowing Jaśkowski's work, da Costa began to develop his ideas about the importance of the study of contradictory theories.

In 1958, in "Notas sobre o conceito de contradição" (Notes on the concept of contradiction, in Portuguese), da Costa ((1958) and (1959)) proposes the following *Principle of Tolerance in Mathematics*:

From the sintactic and semantical point of view, every theory is permitable, since it is not trivial.

From his Thesis ***Sistemas Formais Inconsistentes*** (Inconsistent Formal Systems, in Portuguese) da Costa initiates, in 1963, the publication of a series of papers, in which he introduces his hierarquies of paraconsistent calculi (da Costa (1963a), (1963b), (1964a), (1964b), (1964c), (1965), (1967), (1971), (1974) and (1986)).

In ***Sistemas Formais Inconsistentes***, the objectives of his pionneer work are clearly defined, what gave origin to the area of research of the *paraconsistent non-classical logics*.

It should be mentioned that the adjective 'paraconsistent' was suggested by the Peruvian philosopher F. Miró-Quesada, in 1976.

In 1963, da Costa introduces his hierarchies of logical calculi for the study of inconsistent (contradictory) but non-trivial theories: the hierarchy of propositional calculi $\mathbf{C_n}$, $1 \leq n \leq \omega$, the hierarchy of predicate calculi $\mathbf{C_n}^*$, $1 \leq n \leq \omega$; the hierarchy of predicate calculi with equality $\mathbf{C_n}^=$, $1 \leq n \leq \omega$, and the hierarchy of calculi of descriptions $\mathbf{D_n}$, $1 \leq n \leq \omega$.

From 1963, da Costa has developed several systems related to paraconsistency and he apparently "became the first logician to develop strong logical systems involving contradictions which could be useful for substantive parts of mathematics as well as the empirical and human sciences" (see da Costa, Krause and Bueno (2007)).

Da Costa, his disciples and collaborators, in special Ayda Arruda (see Arruda (1970a), (1970b) and (1975)), from Brazil and several other countries, as for instance Argentina, Australia, Belgium, Bulgaria, Ecuador, Italy, Peru, Polland, Russia and USA, have introduced many paraconsistent systems, having obtained relevant results concerning the decidability, proof theory and algebraic structures associated to such systems; paraconsistent set theories; logics of higher order; model theory; paraconsistent differential calculus; and some applications to computer science, the foundations of science and to technology.

Nowadays 'paraconsistency' can be considered as a field of knowledge.

5. Final Remarks

With the crisis of the paradoxes at the beginning of the 20th Century, the publication of Russell's ***Principia*** and the creation of set theory, Kant's 'finished science' had meaningful transformations, which motivated its development with the creation of several research areas, and under certain circumstances characterized logic also as a discipline of mathematics.

The development of non-classical logics in general has opened several research areas and has propiciated the solution of important questions relative to the foundations of science.

Several applications of many-valued logics have been studied and developed, such as to the theory of electric circuits, to linguistics, to computers programming and to the theory of probability.

Nowadays, many-valued logics have been innovatingly applied to the treatment of information in conditions of uncertainty and to problems there originated, including those concerning to computability and complexity. For this applications the algebraic approach is indispensable (see Cignoli, D'Ottaviano and Mundici (1995) and (2000)).

Relatively to modal logic and its development, dynamic logic deserves to be mentioned, that is, the logic that represents processes of computation. Dynamic logic can be seen as an application of modal logic to informatics and it offers a meaningful example of Kripke's semantics fruitfulness.

However, some of the most significant applications of the modal semantics can be seen in the field of linguistics.

Paraconsistent logic is closely related to several other types of non-classical logics, especially to relevant and dialectical logic, many-valued and intuitionistic logic, fuzzy logic, to the general theory of vagueness and Meinong's theory of objects, as well as to the logical thesis of the 'last' Wittgenstein (see D'Ottaviano (1990) and da Costa, Krause and Bueno (2007)).

The study of paraconsistent logics, besides allowing the construction of paraconsistent theories, makes possible the direct study of the logical and semantic paradoxes, without trying to avoid them; the study of certain principles in all their strength, as for instance the Principle of Comprehension in set theory; and perhaps it permits us a better understanding of the concept of negation.

Among the applications of paraconsistent logic are its use in ethics; doxastic, deontic and epistemic logics; and the theory of probability. Recent works connect paraconsistent logic to the study of semantically closed languages, the foundations of quantum mechanics, artificial intelligence, cognitive sciences, the foundations of infinitesimal calculus and technology (see da Costa, Krause and Bueno (2007), da Costa (2000), D'Ottaviano and Carvalho (2005), Abe and da Silva (1998) and (2003), Akama (2001), Blair and Subrahmanian (1989a) and (1989b), Carnielli, Coniglio and D'Ottaviano (2002) and da Costa, Béziau and Bueno (1998)).

It is usually accepted the Fregue's work was responsable for definitely separating logic of philosophy and of mathematics. The new science, with 'exact' methods as the ones of mathematics and wide interests as the philosophical ones, treated the deep consequences of the important universal simplifications. The logician became responsible for the reorganization of the foundations of mathematics.

With the creation of non-classical logics, there was the possibility of the formalization of universes of discourse more complex than the mathematical domain.

Besides, the improvement of computability theory showed us two fundamental facts: not all the mathematical procedures are computable, and not all the mathematical procedures that are computable can be actually computed. Then, a very relevant question appeared, the rigorous study of the human intelligence and of the cognitive processes, the human decision process and the creation process.

Besides the new and not foreseen uses of classical logic, it is not difficult to perceive the connexion between non-classical logics and artificial intelligence and cognitive sciences, that has among its interests the reasoning processes that can be formulated and controlled in the computable mathematical universe and, so, must be naturally based on logic.

Paraconsistent logics, looked as formal theories that support inconsistent but non-trivial theories, constitute a natural solution for the treatment of the question of tolerance to fails: an intelligent system has to work under imprecision of language, of all kinds of especifications, and inclusive under imprecision of consistency.

In 1986, Mikemberg, da Costa and Chuaqui introduced a formal definition of pragmatic truth, latter called *quasi-truth* by da Costa, on trying to capture the meaning of the theories of pragmatist thinkers such as Peirce and James. This concept of quasi-truth is considered by da Costa as the truth conception inherent to empirical theories and is a generalization (for partial contexts) of Tarski's correspondence characterization of truth. In D'Ottaviano and Hifume (2007) we have analysed a suitable paraconsistent modal logic that can be used as the underlying logic for theories whose truth conception is the quasi-truth.

Carnielli, Coniglio and Marcos (2007) present a new and creative approach to the study of paraconsistent logics.

D'Ottaviano and Feitosa (2007) discuss a research project, developed during several years, which possibilitated them the analysis of inter-relations between several paraconsistent logics and classical logic and between paraconsistent logic and other non-classical logics, through the study of *translations* between them (see D'Ottaviano and Feitosa (2000)).

Another recent and very interesting approach to understanding inter-relations between paraconsistent logics and other kinds of logics, is through *combinations* (fibring and splicing) of logics (see Carnielli et all (2007)).

We are living a new revolution of logic, with the emergence of new paradigms – connected to the creation of computers, robots and of new areas of science.

We are not derogating the classical Aristotelian logic, contrarily, we of course know the enormous gama of situations whose analysis explicitly depends on it.

But, from the arising of non-classical logics, and with the new paradigm that they conjecture for the 21th Century, we know that there is not "one" logic, but a better and more adequate logic for every type of problem.

We conclude with da Costa's words:

"Nowadays, logic is one of the most exciting branchs of knowledge... and one of the greatest cultural revolutions of our epoch was the construction of non-classical logics, particularly of the heterodox logics, such a revolution similar to that provoqued by the discovering of the non-Euclidean geometries, in the 19th Century."

References

ABE, J. M., DA SILVA FILHO, J. I. Inconsistency and electronic circuits, *Proceedings of the International ICSC Symposium on Engineering of Intelligent Systems (EIS'98)*, v. 3, *Artificial Intelligence*, Alpaydin, E. (ed.), ICSC Academic Press International Computer Science Conventions Canada-Switzerland, pp. 191-197, 1998.

ABE, J. M., DA SILVA FILHO, J. I. Manipulating conflicts and uncertainties in robotics, *Multiple-Valued Logic and Soft Computing*, v. 9, pp. 147-169, 2003.

AKAMA, S., ABE, J. M. The role of inconsistency in information systems, *Proc. of the 5th World Multiconference on Systemics, Cybernetics and Informatics (SCI'2001)*, Orlando, USA, pp. 355-360, 2001.

ARISTOTLE. De interpretatione. Trad. E. M. Edghill. In: HUTCHINS, R. M. (ed.). **Great books of the western world**. Chicago: Encyclopaedia Britannica, v. 8, pp. 23-36, 1978a.

————. Analytica priora. Trad. E. M. Edghill. In: HUTCHINS, R. M. (ed.). **Great books of the western world**. Chicago: Encyclopaedia Britannica, v. 8, pp. 37-93, 1978b.

ARRUDA, A. I. Sur les systèmes formels NF_i de da Costa. *Comptes Rendus de l'Académie des Sciences de Paris*, t.270A, p. 1081-1084, 1970a.

————. Sur le système NF_ω. *Comptes Rendus de l'Académie des Sciences de Paris*, t.270A, p. 1137-1139, 1970b.

————. Le schéma de la séparation dans les systèmes NF_n. *Comptes Rendus de l'Académie de Sciences de Paris*, t.280A, p. 1341-1344, 1975.

————. On the imaginary logic of N. A. Vasil'év, in A. I. Arruda, N. C. A. da Costa and R. Chuaqui (eds.), **Non-Classical Logics, Model Theory and Computability**, North Holland, pp. 3-22, 1977.

————. A survey of paraconsistent logic. In: LATIN AMERICAN SYMPOSIUM ON MATHEMATICAL LOGIC, 4., 1978, Santiago, Chile. **Mathematical Logic in Latin America:** proceedings. A. I. Arruda, N. C. A. da Costa e R. Chuaqui (eds.). Amsterdam: North Holland, pp. 1-41, 1980.

————. Aspects of the historical development of paraconsistent logic. In: G. Priest, R. Routley and J. Norman (eds.). **Paraconsistent Logic:** essays on the inconsistent. Philosophia Verlag, 1989.

————. *N. A. Vasiliev e a Lógica Paraconsistente*. Campinas: COLEÇÃO CLE, v. 7, 1992.

ASENJO, F. G. A calculus of antinomies, *Notre Dome Journal of Formal Logic* **XVI**, pp. 103-105, 1966.

Atti del IV Congresso Internazionale del Matematici, v. 1, Roma, 1909.

BLAIR, H. A., SUBRAHMANIAN, V. S. Paraconsistent Logic Programming, *Theoretical Computer Science* 68, pp. 135-154, 1989a.

————. Paraconsistent foundations of logic programming, *The Journal Non-Classical Logic* **5**, pp. 44-73, 1989b.

BOBENRIETH MISERDA, A. **Inconsistencias, ?por que no?**, Bogotá: Cocultura, 1996.

BOCHENSKI, I. M. **A history of formal logic.** New York: Chelsea Publishing Co., 1961.

BOCHVAR, D. On a three valued logical calculus and its applications to the analysis of contradictions. Mathematičéskij Sbornik (Recueil Mathematiques, 4, pp. 287-300, 1939.

BOOLE, G. **An investigation of the laws of thought.** New York: Dover, 1958. (Publicado originalmente em 1847)

BORKOWSKI, L. **Selected works of J. Łukaziewicz.** Amsterdam: North Holland, 1970.

CANTOR, G. Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen. J. Reine Angew. *Mathematische Annalen.*, v. 77, pp. 258-262, 1874.

CARNIELLI, W. A., CONIGLIO, M. E., D'OTTAVIANO, I. M. L. **Paraconsistency: the logical way to the inconsistent**, Proceedings of the Second World Congress on Paraconsistency. Marcel Dekker: 2002.

CIGNOLI, R. L. O., D'OTTAVIANO, I. M. L., MUNDICI, D. *Algebraic Foundations of Many-valued Reasoning*. Dordrecht: Kluwer Academic Published, *Trends in Logic*, v. 2, 2000.

————. *Álgebras das Lógicas de Łukasiewicz*. Campinas: COLEÇÃO CLE, v. 12 (2ª Ed.), 1995.

COUTURAT, L. (ed.). **Opuscules et fragments inédits de Leibniz.** Paris: 1903.

DA COSTA, N. C. A. **Sistemas formais inconsistentes.** Thesis, Universidade Federal do Paraná: 1963a.

————. Calculs propositionels pour les systèmes formels inconsistants. *Comptes Rendus de l'Académie de Sciences de Paris*, t.257, pp. 3790-3793, 1963b.

————. Calculs de prédicats pour les systèmes formels inconsistants. *Comptes Rendus de l'Académie de Sciences de Paris*, t.258, pp. 27-29, 1964a.

————. Calculs de prédicats avec égalité pour les systèmes formels inconsistants. *Comptes Rendus de l'Académie de Sciences de Paris*, t.258, pp. 1111-1113, 1964b.

————. Sur un système inconsistant de théorie des ensembles. *Comptes Rendus de l'Académie de Sciences de Paris*, t.258, pp. 3144-3147, 1964c.

————. Sur les systèmes formels C_1, C_1^*, $C_1^=$, D_i et NF_i. *Comptes Rendus de l'Académie de Sciences de Paris*, t.260, pp. 5427-5430, 1965.

————. Une nouvelle hiérarchie de théories inconsistantes. *Publications du Département de Mathématiques*. Université de Lyon, n. 4, pp. 2-8, 1967.

————. Remarques sur le système NF_1. *Comptes Rendus de l'Académie de Sciences de Paris*, t.272A, pp. 1149-1151, 1971.

————. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, v. 15, n. 4, pp. 497-510, 1974.

————. On paraconsistent set theory. *Logique et Analyse*, v. 115, pp. 361-371, 1986.

————. Paraconsistent mathematics. In: I WORLD CONGRESS ON PARACONSISTENCY, 1998, Ghent, Belgium. **Frontiers in Paraconsistent Logic:** proceedings. D. Batens, C. Mortensen, G. Priest, J. P. van Bendegen (eds.). London: King's College Publications, pp. 165-179, 2000.

DA COSTA, N. C. A., BÉZIAU, J. Y., BUENO, O. **Elementos de teoria paraconsistente de conjuntos**. Campinas: Unicamp/CLE, 1998. (Coleção CLE, v. 23)

DA COSTA, N. C. A., MARCONI, D. An overview of paraconsistent logics in the 80's. *The Journal of Non-Classical Logic*, v. 6, n. 1, pp. 5-31, 1989.

DE MORGAN, A. **Formal logic:** or the calculus of inference, necessary and probable. London: Taylor & Walton, 1847.

D'OTTAVIANO, I. M. L. On the development of paraconsistent logic and da Costa's work. *The Journal of Non-Classical Logic*, v. 7 (1/2), pp. 89-152, 1990.

D'OTTAVIANO, I. M. L., CARVALHO, T. F. Sobre o infinitésimo e o cálculo diferencial paraconsistente de da Costa. *Revista Eletrônica Informação e Cognição*, v. 4, pp. 78-102, 2005.

D'OTTAVIANO, I. M. L., FEITOSA, H. A. Paraconsistent logics and translations. *Synthese*, v. 125, n. 1/2, pp. 77-95, 2000.

————. História da lógica e o surgimento das lógicas não-clássicas. In: NOBRE, Sérgio (org.). **Coleção História da Matemática para Professores**. Rio Claro: Sociedade Brasileira de História da Matemática, pp. 1-66, 2003.

————. Deductive systems and translations. In: BÉZIAU, J.-Y., COSTA-LEITE, A. (Eds.), *Perspectives on Universal Logic*. Monza: Polimetrica International Scientific Publisher, pp. 127-157, 2007.

D'OTTAVIANO, I. M. L., HIFUME, C. Peircean pragmatic truth and da Costa's quase-truth. Studies in Computational Intelligence, 64, pp. 383-398, 2007.

FREGE, G. **Begriffsschriff**: eine der arithmetischen nach gebildete Formelsprache des reinen Denkens. Hildesheim: Georg Olms Verlag, 1977. Originally published in 1879).

GUILLAUME, M. Regard en arrière sur quinze annés de coopératioli douce avec l'écolebrézilienne de logique paraconsistante. *Logique et Analyse* **153-154**, pp. 5-14, 1996.

HAACK, S. *Deviant Logic*, Cambridge University Press, 1974.

JAŚKOWSKI, S. Rachunek zdan dla systemów dedukcyjnych sprzecznych, *Studia Societatis Scientiarum Torunensis*, Sectio A, I, pp. 55-77, 1948.

————. O konjuncji dyskusyjnej w rachunsku zdán dla systemów dedukcyjnych sprzeczcych, *Studia Societatis Torunensis*, Sectio A, I, pp. 171-172, 1949.

————. Propositional calculus for contradictory deductive systems, *Studia Logica*, n. 24, p. 143-157, 1969.

KANT, I. **Kritik der reinem Vernunft**. Riga : Johanen Friedrich Hartknoch, 1787.

KLEENE, S. C. **Introduction to metamathematics**. Amsterdam: North Holland; New York: Van Nostrand, 1952.

LEIBNIZ, G. W. **Logical papers**. Trad. G. H. R. Parkinson. Oxford: Clarendon Press, 1966.

ŁUKASIEWICZ, J. **O Zasadzie sprzecznosci u Arystotelesa: studium krytyczne**. Kraków: Akademia Umiejetnosci, 1910.

————. Über den satz von widerspruchs bei Aristoteles. *Bulletin International de l'Académie des Sciences de Cracovie*: Classe d'Histoire de Philosophie, pp. 15-38, 1910. Translation into English in **Borkowski 1970**.

MIKENBERG, I., DA COSTA, N. C. A., CHUAQUI, R. Pragmatic truth and approximation to truth, *The Journal of Symbolic Logic* **51**, pp. 201-221, 1986.

NAKAMATSU, K., ABE, J. M., SUZUKI, A. 'Defeasible deontic robot control based on extended vector annotated logic programming', *Computer Antecipatory Systems: (SASYS 2001) – Fifth International Conference*, D. M. Dubois (ed.), American Institute of Physics, pp. 490-500, 2002.

NELSON, D. Constructive falsity, *J. Symb. Logic* **14**, pp. 16-26, 1949.

PEIRCE, C. S. **Philosophical Writings of Peirce**, selected and edited by Buchler, J., New York, Dover, 1965.

POST, E. L. Introduction to a general theory of elementary propositions. *American Journal of Mathematics*, v. 43, pp. 163-185, 1921.

SCHOENFIELD, J. R. **Mathematical Logic**. Reading: Addison-Wesley, 1967.

WHITEHEAD, A. N., RUSSELL, B. **Principia mathematica**. Cambridge: Cambridge University Press, 1910-1913.